

# **Use of GRA-Fuzzy and TOPSIS for Multi-Response Optimization in CNC End Milling**

Thesis submitted in partial fulfillment of the requirements for the Degree of

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In

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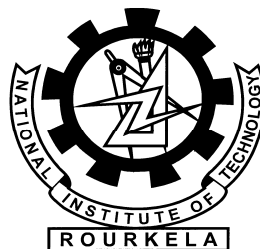
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**Certificate of Approval**

This is to certify that the thesis entitled **USE OF GRA-FUZZY AND TOPSIS FOR MULTI-RESPONSE OPTIMIZATION IN CNC END MILLING** submitted by **Sri Bikash Mohanty** has been carried out under my supervision in partial fulfillment of the requirements for the Degree of **Bachelor of Technology** in **Mechanical Engineering** at National Institute of Technology, Rourkela, and this work has not been submitted elsewhere before for any other academic degree/diploma.

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## Abstract

In production/manufacturing context, multi-response optimization of machining processes is one of the most important areas of research to find out the best process environment for any machining operation. Literature is seemed rich in addressing multi-response optimization problems using various techniques. It has been viewed that these methods are not efficient enough due to so many assumptions and limitations imposed upon it. Therefore, researchers are concentrating in hybridizing various methods to empower advantageous aspects thereby avoiding/overcoming inherent limitations of aforesaid individual methodologies. The present study aimed to develop such a hybrid method which could efficiently be applied for continuous quality improvement for a process/product and to facilitate in off-line quality control of any manufacturing process. In the present work, two optimization philosophies (i) *TOPSIS based Taguchi method* and (ii) *Grey relation analysis (GRA) followed by Fuzzy logic and Taguchi method* has been proposed here to optimize machining parameters in CNC end milling operation towards improving quality as well as productivity simultaneously.

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## **1. Introduction: Overview of CNC Machining**

As far as machining processes are concerned CNC has evolved over the conventional machine tools. Some of the advantages of CNC machine tool over conventional machine tool are listed below:

- 1) Consistency of work pieces produced- Since a CNC machine executes a program, and it will do so in exactly the same fashion time and time again, the consistency of work pieces produced is much better than work pieces run on conventional machine tools.
- 2) Faster work piece machining- Since current model CNC machine tools are guarded (splash guards, windows, etc.) in a much better manner than most conventional machine tools, users can apply the most efficient cutting conditions to attain the best cycle times. Manual machinists tend to nurse-along their machining operations to minimize the chips and coolant is constantly thrown from the work area.
- 3) Lowered skill level of machinist- Though there are some misconceptions in this area (some people believe that anyone can run CNC machines without training), the level of skill required to run (but not program) a CNC machine is much lower than that required to run a conventional machine tool - especially in a production environment when the same work piece is run over and over again.
- 4) Complexity of work pieces to be machined- CNC machines can generate very complex motions, making it possible to machine shapes that cannot be generated (or are extremely difficult to generate) on conventional machine tools.

- 5) Flexibility, faster turn-around, and smaller lots- Because they're programmable, a given CNC machine can be used to machine a large variety of different work pieces. Most are also designed to minimize downtime between production runs (setup time). Some conventional machines they're replacing (screw machines and transfer lines, for example) are extremely difficult to setup, making them feasible only for larger lot sizes.

In CNC end milling, which is the area of focus in the present work, the various process parameters are involved listed as follows:

- 1) Cutting speed (also called surface speed or simply speed) is the speed difference (relative velocity) between the cutting tool and the surface of the work piece it is operating on. It is expressed in units of distance along the work piece surface per time, typically surface feet per minute (sfm) or meters per minute (m/min).
- 2) Feed rate (feed) is the relative velocity at which the cutter is advanced along the work piece; its vector is perpendicular to the vector of cutting speed. Feed rate units depend on the motion of the tool and work piece; in rotating systems (e.g., turning and boring), the units are almost always distance per spindle revolution (inches per revolution [in/rev or ipr] or millimeters per revolution [mm/rev]).
- 3) Depth of cut - The depth of cut (DOC) is the distance that a tool penetrates into the work piece. It is generally measured in mm.

The output responses of the CNC milling process can be measured in terms of various surface roughness characteristics of statistical importance (of the CNC end milled surface), Material Removal Rate (MRR), dimensional accuracy, tool (cutter) life-tool wear, extent of chatter and vibration and many others. Appropriate selection as well as

precise control of process parameters can yield desired level of product quality with increased productivity.

## **2. State of Art Understanding and Problem Formulation**

**Ghani et al. (2004)** outlined the Taguchi optimization methodology, which was applied to optimize cutting parameters in end milling when machining hardened steel AISI H13 with TiN coated P10 carbide insert tool under semi-finishing and finishing conditions of high speed cutting. The milling parameters evaluated was cutting speed, feed rate and depth of cut. An orthogonal array, signal-to-noise (S/N) ratio and Pareto analysis of variance (ANOVA) were employed to analyze the effect of these milling parameters.

**Wattanuchariya and Pintasee (2006)** attempted to optimize the metallic milling parameters for surface finishing. The two controlled parameters were spindle speed and feed rate. Three materials: aluminum, brass and cast iron were tested. The research methodology concerned the Response Surface Methodology (RSM) by Central Composite Design (CCD). Then, the Al 2072, brass with 10% zinc and cast iron (A287) were tested in order to investigate the relationship between the controlled parameters.

**Gopalsamy (2009)** applied Taguchi method to find optimal process parameters for end milling while hard machining of hardened steel. An orthogonal array, signal-to-noise ratio and ANOVA were applied to study performance characteristics of machining parameters (cutting speed, feed, depth of cut and width of cut) with consideration of surface finish and tool life. Chipping and adhesion were observed to be the main causes of wear. Multiple regression equations were formulated for estimating predicted values of surface roughness and tool wear.



**Ginta et al. (2009)** focused on developing an effective methodology to determine the performance of uncoated WC-Co inserts in predicting minimum surface roughness in end milling of titanium alloys Ti-6Al-4V under dry conditions. Central composite design of response surface methodology was employed to create an efficient analytical model for surface roughness in terms of cutting parameters: cutting speed, axial depth of cut, and feed per tooth.

**Ab. Rashid et al. (2009)** presented the development of mathematical model for surface roughness prediction before milling process in order to evaluate the fitness of machining parameters; spindle speed, feed rate and depth of cut. Multiple regression method was used to determine the correlation between a criterion variable and a combination of predictor variables. It was established that the surface roughness was most influenced by the feed rate.

**Alwi (2010)** studied the optimum of surface roughness by using response surface method. The experiments were carried out using CNC milling machine. All the data was analyzed by using Response Surface Method (RSM) and Neural Network (NN). The result showed that the feed gave the more affect on the both prediction value of Ra compare to the cutting speed and depth of cut.

**Routara et al. (2010)** highlighted a multi-objective optimization problem by applying utility concept coupled with Taguchi method through a case study in CNC end milling of UNS C34000 medium leaded brass.

**Patwari et al. (2011)** described mathematically the effect of cutting parameters on surface roughness in end milling of Medium Carbon Steel. The mathematical model for the surface roughness was developed, in terms of cutting speed, feed rate, and axial depth

of cut. The effect of these cutting parameters on the surface roughness was carried out using design of experiments and response surface methodology (RSM).

In order to solve a multi-objective optimization problem, it is recommended to convert multiple objectives into a single representative objective function. This is to be optimized (maximized/minimized) finally using any optimization algorithm/philosophy. In product/process optimization Taguchi method is very popular as it selects optimal solution (parameter setting) in discrete points in the parameter domain. But this approach fails to solve multi-response optimization problem. In this context, application of TOPSIS adapted from Multi-Criteria Decision Making (MCDM) and GRA-Fuzzy deserves mention. Aforesaid methodologies help to convert multi response parameters into single response. The study exhibits that application of grey-fuzzy has been found more advantageous over TOPSIS method. In fuzzy-based approach priority weights of individual responses need not to be assigned by decision-makers. Fuzzy inference engine can tackle this aspect in its internal hierarchy. This is the main advantage of using fuzzy expert system over conventional optimization tools.

### **3. Outline of Taguchi Method**

Taguchi's philosophy is an efficient tool for the design of high quality manufacturing system. *Dr. Genichi Taguchi*, a Japanese quality management consultant, developed a method based on Orthogonal Array (OA) of experiments, which provided much-reduced variance for the experiment with optimum setting of process control parameters. Thus the integration of Design Of Experiments (DOE) with parametric optimization of process is achieved in the Taguchi Method, which would yield desired results. Orthogonal Array

(OA) provides a set of well-balanced experiments (with less number of experimental runs), and Taguchi's signal-to-noise ratios (S/N), which are logarithmic functions of desired output; serve as objective functions in the optimization process. This technique helps in data analysis and prediction of optimum results. In order to evaluate optimal parameter settings, Taguchi Method uses a statistical measure of performance called signal-to-noise ratio. The S/N ratio takes both the mean and the variability into account. The S/N ratio is the ratio of the mean (Signal) to the standard deviation (Noise). The ratio depends on the quality characteristics of the product/process to be optimized. The standard S/N ratios generally used are as follows: - Nominal is Best (NB), Lower the Better (LB) and Higher the Better (HB). The optimal setting is the parameter combination, which has the highest S/N ratio. The steps involved in Taguchi Method are as follows:

**Step 1:** Formulation of the problem: the success of an experiment depends on complete understanding of the nature of the problem. Identification of the performance characteristic of the process output is most important in connection with application of Taguchi method.

**Step 2:** Identification of control factors, noise factors and signal factors: A controlled factor is a characteristic that can be controlled in the product or process subjected to designing. Noise factors are those that cannot be easily controlled in the manufacture or use of a product. In the experimental setting, the levels of noise factors are to be controlled for simulating the sources of variation the product will be subjected to in actual use. The goal of robust parameter design is to find levels of the control factors that will minimize the sensitivity of the product to changes in the noise factors. A signal

factor is an input to the experimental system that is supposed to affect the output. Taguchi's dynamic experiment measures the response variable at different levels of a signal factor.

**Step 3:** Selection of factor levels, possible interactions and degrees of freedom associated with each factor and the interaction effects: Experimental domain has to be selected first with different levels of factors. Main/direct effects as well as interaction effects of the factors are to be selected to incorporate in experimental design accordingly.

**Step 4:** Design of an appropriate Orthogonal Array (OA): Taguchi's orthogonal arrays are experimental designs that usually require only a fraction of the full factorial combinations. The arrays are designed to handle as many factors as possible in a certain number of runs as that dictated by full factorial design. The columns of the arrays are balanced and orthogonal. This means that in each pair of columns, all factor combinations appear the same number of times. Orthogonal designs allow estimating the effect of each factor on the response independently of all other factors.

**Step 5:** Experimentation and data collection: Experiments are to be conducted and collected data are to be utilized for analysis of the process towards process optimization.

**Step 6:** Statistical analysis and interpretation of experimental results: Evaluation of statistical significance of the factors on the selected response variable is done in this step and interpretation is made based on this evaluation.

**Step 7:** Conducting confirmatory test: Taguchi's predicted optimal result can be verified by this test.

## 4. TOPSIS Method

TOPSIS (*technique for order preference by similarity to ideal solution*) method was firstly proposed by (Hwang and Yoon, 1981). The basic concept of this method is that the chosen alternative (appropriate alternative) should have the shortest distance from the positive ideal solution and the farthest distance from negative ideal solution. Positive ideal solution is a solution that maximizes the benefit criteria and minimizes adverse criteria, whereas the negative ideal solution maximizes the adverse criteria and minimizes the benefit criteria. The steps involved for calculating the TOPSIS values are as follows:

**Step 1:** This step involves the development of matrix format. The row of this matrix is allocated to one alternative and each column to one attribute. The decision making matrix can be expressed as:

$$\mathbf{D} = \begin{matrix} & \begin{matrix} A_1 \\ A_2 \\ \vdots \\ A_i \\ \vdots \\ A_m \end{matrix} \end{matrix} \begin{bmatrix} x_{11} & x_{12} & \cdot & x_{1j} & x_{1n} \\ x_{21} & x_{22} & \cdot & x_{2j} & x_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ x_{i1} & x_{i2} & \cdot & x_{ij} & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ x_{m1} & x_{m2} & \cdot & x_{mj} & x_{mn} \end{bmatrix} \quad (1)$$

Here,  $A_i$  ( $i=1, 2, \dots, m$ ) represents the possible alternatives;  $x_j$  ( $j=1, 2, \dots, n$ ) represents the attributes relating to alternative performance,  $j=1, 2, \dots, n$  and  $x_{ij}$  is the performance of  $A_i$  with respect to attribute  $X_j$ .

**Step 2:** Obtain the normalized decision matrix  $r_{ij}$ . This can be represented as:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^m x_{ij}^2}} \quad (2)$$

Here,  $r_{ij}$  represents the normalized performance of  $A_i$  with respect to attribute  $X_j$ .

**Step 3:** obtain the weighted normalized decision matrix,  $\mathbf{V} = [v_{ij}]$  can be found as:

$$V = w_j r_{ij} \quad (3)$$

Here,  $\sum_{j=1}^n w_j = 1$

**Step 4:** Determine the ideal (best) and negative ideal (worst) solutions in this step. The ideal and negative ideal solution can be expressed as:

a) The ideal solution:

$$\begin{aligned} A^+ &= \left\{ \left( \max_i v_{ij} \mid j \in J \right), \left( \min_i v_{ij} \mid j \in J' \mid i = 1, 2, \dots, m \right) \right\} \\ &= \{v_1^+, v_2^+, \dots, v_j^+, \dots, v_n^+\} \end{aligned} \quad (4)$$

b) The negative ideal solution:

$$\begin{aligned} A^- &= \left\{ \left( \min_i v_{ij} \mid j \in J \right), \left( \max_i v_{ij} \mid j \in J' \mid i = 1, 2, \dots, m \right) \right\} \\ &= \{v_1^-, v_2^-, \dots, v_j^-, \dots, v_n^-\} \end{aligned} \quad (5)$$

Here,

$J = \{j = 1, 2, \dots, n \mid j\}$ : Associated with the beneficial attributes

$J' = \{j = 1, 2, \dots, n \mid j\}$ : Associated with non beneficial adverse attributes

**Step 5:** Determine the distance measures. The separation of each alternative from the ideal solution is given by n-dimensional Euclidean distance from the following equations:

$$S_i^+ = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^+)^2}, \quad i = 1, 2, \dots, m \quad (6)$$

$$S_i^- = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^-)^2}, i = 1, 2, \dots, m \quad (7)$$

**Step 6:** Calculate the relative closeness to the ideal solution:

$$C_i^+ = \frac{S_i^-}{S_i^+ + S_i^-}, i = 1, 2, \dots, m; 0 \leq C_i^+ \leq 1 \quad (8)$$

**Step 7:** Rank the preference order. The alternative with the largest relative closeness is the best choice.

In the present study  $C_i^+$  for each product has been termed as Multi-Performance Characteristic Index (MPCI) which has been optimized by Taguchi method.

## 5. Grey Relational Analysis

In grey relational analysis, experimental data i.e. measured features of quality characteristics are first normalized ranging from zero to one. This process is known as grey relational generation. Next, based on normalized experimental data, grey relational coefficient is calculated to represent the correlation between the desired and actual experimental data. Then overall grey relational grade is determined by averaging the grey relational coefficient corresponding to selected responses. The overall performance characteristic of the multiple response process depends on the calculated grey relational grade. This approach converts a multiple- response- process optimization problem into a single response optimization situation, with the objective function is overall grey relational grade. The optimal parametric combination is then evaluated which would result highest grey relational grade. The optimal factor setting for maximizing overall grey relational grade can be performed by Taguchi method.

In grey relational generation, the normalized bead width, reinforcement and HAZ width, corresponding to lower-the-better (LB) criterion can be expressed as:

$$x_i(k) = \frac{\max y_i(k) - y_i(k)}{\max y_i(k) - \min y_i(k)} \quad (9)$$

Bead penetration and %Dilution should follow larger-the-better criterion, which can be expressed as:

$$x_i(k) = \frac{y_i(k) - \min y_i(k)}{\max y_i(k) - \min y_i(k)} \quad (10)$$

Here  $x_i(k)$  is the value after the grey relational generation,  $\min y_i(k)$  is the smallest value of  $y_i(k)$  for the  $k$ th response, and  $\max y_i(k)$  is the largest value of  $y_i(k)$  for the  $k$ th response. An ideal sequence is  $x_0(k)$  ( $k = 1, 2, 3, \dots, 9$ ) for the responses. The definition of grey relational grade in the course of grey relational analysis is to reveal the degrees of relation between the sequences say,  $[x_0(k) \text{ and } x_i(k), i = 1, 2, 3, \dots, 9]$ . The grey relational coefficient  $\xi_i(k)$  can be calculated as:

$$\xi_i(k) = \frac{\Delta_{\min} + \psi \Delta_{\max}}{\Delta_{0i}(k) + \psi \Delta_{\max}} \quad (11)$$

Here  $\Delta_{0i} = \|x_0(k) - x_i(k)\|$  = difference of the absolute value  $x_0(k)$  and  $x_i(k)$ ;  $\psi$  is the distinguishing coefficient  $0 \leq \psi \leq 1$ ;  $\Delta_{\min} = \forall j^{\min} \in i \forall k^{\min} \|x_0(k) - x_j(k)\|$  = the smallest value of  $\Delta_{0i}$ ; and  $\Delta_{\max} = \forall j^{\max} \in i \forall k^{\max} \|x_0(k) - x_j(k)\|$  = largest value of  $\Delta_{0i}$ . After averaging the grey relational coefficients, the grey relational grade  $\gamma_i$  can be computed as:



$$\gamma_i = \frac{1}{n} \sum_{k=1}^n \xi_i(k) \quad (12)$$

Here  $n$  = number of process responses. The higher value of grey relational grade corresponds to intense relational degree between the reference sequence  $x_0(k)$  and the given sequence  $x_i(k)$ . The reference sequence  $x_0(k)$  represents the best process sequence. Therefore, higher grey relational grade means that the corresponding parameter combination is closer to the optimal. The mean response for the grey relational grade with its grand mean and the main effect plot of grey relational grade are very important because optimal process condition can be evaluated from this plot.

## 6. Fuzzy Inference System (FIS)

Fuzzy logic is a superset of conventional (boolean) logic that has been extended to handle the concept of partial truth, where the truth value may range between completely true and completely false. A fuzzy inference system (FIS) defines a nonlinear mapping of the input data vector into a scalar output, using fuzzy rules. A fuzzy rule based system consists of four parts:

1. *knowledge base,*
2. *fuzzifier,*
3. *inference engine and*
4. *defuzzifier.*

**Fuzzifier:** The real world input to the fuzzy system is applied to the fuzzifier. In fuzzy literature, this input is called crisp input since it contains precise and specific information

about the parameter. The fuzzifier convert this precise quantity to the form of imprecise quantity like ‘large’, ‘medium’, ‘high’ etc. with a degree of belongingness to it. Typically the value ranges from 0 to 1.

**Knowledge base:** The main part of the fuzzy system is the knowledge base in which both rule base and database are jointly referred. The database defines the membership functions of the fuzzy sets used in the fuzzy rules whereas the rule base contains a number of fuzzy IF – THEN rules.

**Inference engine:** The inference system or the decision making input perform the inference operations on the rules. It handles the way in which the rules are combined.

**Defuzzifier:** The output generated by the inference block is always fuzzy in nature. A real world system will always require the output of the fuzzy system to the crisp or in the form of real world input. The job of the defuzzifier is to receive the fuzzy input and provide real world output. In operation, it works opposite to the input block.

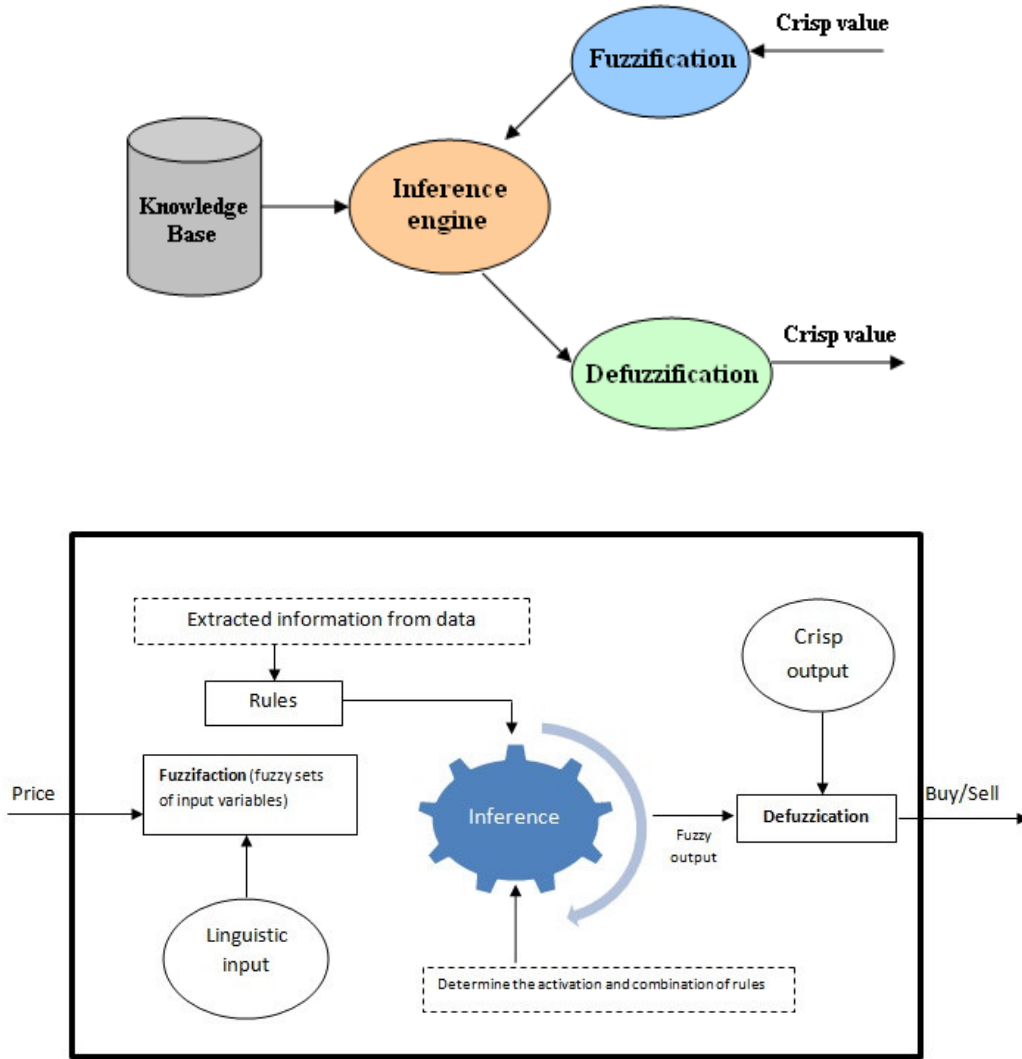
The first step in system modeling was the identification of input and output variables called the system variables. In the selection procedure, the inputs and the outputs are taken in the form of linguistic format. A linguistic variable is a variable whose values are words or sentences in natural or man-made languages. Linguistic values are expressed in the form of fuzzy sets. A fuzzy set is usually defined by its membership functions. In general, triangular or trapezoidal membership functions are used to the crisp inputs because of their simplicity and high computational efficiency. In the present study, a fuzzy set  $\tilde{A}$  is represented by triangular fuzzy number which is defined by the triplet  $(a, b, c)$ . Membership function  $\mu_{\tilde{A}}(x)$  is defined as:

$$\forall - x, - a, - b, - c \in R$$

$$\mu_{\tilde{A}}(x) = 0, \text{ if } x < a \text{ else } \left( \frac{x-a}{b-a} \right), \text{ if } a \leq x \leq b \text{ else } \left( \frac{c-x}{c-b} \right), \text{ if } b \leq x \leq c \text{ else } 0, \text{ if } x > c$$

Using a defuzzification method, fuzzy values can be obtained into one single crisp output value. The centre of gravity, one of the most popular methods for defuzzifying fuzzy output functions, is employed in the study. The formula to find the centroid of the

$$\text{combined outputs: } \hat{y}_i = \frac{\int y_i \mu_{ci}(y_i) dy}{\int \mu_{ci}(y_i) dy} \quad (13)$$



**Fig. 1: Elements of Fuzzy Inference System (FIS)**

## 7. Experimentation

Samples of copper bars ( $\varnothing 25 \times 10 \text{ mm}$ ) have been used as work material. Taguchi's  $L_9$  orthogonal array has been used here (**Table 1**). **Table 2** indicates selected process control parameters and their limits. Three machining parameters: cutting speed, feed rate and depth of cut has been varied into three different levels have been used to optimize the machining conditions. HSS tool (C00662D, 12 HSS, TYPE A & N) has been used during experiments. Milling has been performed in CNC MAXMILL set up. Corresponding to each experimental run MRR and average surface roughness values ( $R_a$ ) have been computed (**Table 3**). The surface roughness has been measured by the Talysurf (Taylor Hobson, Surtronic 3+).

## 8. Data Analysis: GRA-Fuzzy combined with Taguchi Method

The methodology used for the optimization is grey relational analysis (GRA) coupled with fuzzy inference system. Grey relational analysis has been utilized to compute grey relation coefficients for individual responses. These have been fed to a Fuzzy Inference System (FIS) as inputs; whose output has been defined as Multi-Performance Characteristic Index (MPCI). MPCI has been optimized finally by Taguchi method. Taguchi method has been used to find an optimal solution at some discrete points at the experimental domain which can be easily adjusted in CNC machine. It is based on two principle namely quadratic quality loss function and Signal-to-Noise (S/N) ratio. The loss function has been used to measure the process response deviating from the desired value and the value of the loss function has been further transformed into an S/N ratio.

Data analysis has been carried out by the procedural hierarchy as shown below.

1. Experimental data (Table 3) have been normalized first (**Table 4**) which is known as grey relational generation.
2. Computation of grey relational coefficients for individual responses in all experimental run (**Table 6**) by considering quality loss estimates of individual responses (**Table 5**). For calculating grey relational coefficients of MRR, a Higher-the-Better (HB) criterion and for  $R_a$ , a Lower-the-Better (LB) criterion has been selected.
3. These grey relational coefficients corresponding to individual responses have been fed as inputs to a Fuzzy Inference System (FIS) (**Fig. 1**). For each of the input parameters seven triangular type membership functions (MFs) have been chosen as: Very Low (VL), Low (L), Fairly Low (FL), Medium (M), Fairly High (FH), High (H) and Very High (VH) (**Fig. 2-3**). Based on fuzzy association rule mapping (**Table 7**) FIS combined multiple inputs into a single output termed as Multi-Performance Characteristic Index (MPCI). The linguistic valuation of MPCI has been represented by seven triangular type membership functions (MFs) have been chosen as: Very Low (VL), Low (L), Fairly Low (FL), Medium (M), Fairly High (FH), High (H) and Very High (VH) (**Fig. 4**). These linguistic values haven transformed into crisp values by defuzzification method.
4. The crisp values of MPCI (**Table 8**) have been optimized by using Taguchi' philosophy. The predicted optimal setting has been evaluated from *Mean (S/N ratio) Response Plot* of MPCIs (**Fig. 5**) and it became A3 B3 C2 D1.
5. Optimal setting has been verified by confirmatory test.

6. Mean response table of S/N ratios of MPCIs has been found in **Table 9**.

## 9. Data Analysis: TOPSIS based Taguchi Method

In TOPSIS based Taguchi approach, experimental data have been normalized first. The normalized data have been furnished in **Table 10**. Elements of normalized decision-making matrix have been multiplied with corresponding response weights to obtain weighted normalized decision-making matrix shown in **Table 11**. Computed *Ideal* and *Negative-Ideal* solutions have been furnished in **Table 12**. Computed distance measures:  $S^+$  and  $S^-$  have been tabulated in **Table 13**. Closeness Coefficient (CC) against each experimental run has been calculated and shown in **Table 14**. CC has been optimized (maximized) finally using Taguchi method. **Fig. 6** reveals S/N ratio plot of closeness coefficient values. Predicted optimal parameter combination has been verified by confirmatory test. Ranking of factors according to their influence on CC has been shown in **Table 15** (mean response table for S/N ratio of CCs).

## 10. Conclusions

This paper outlines application of TOPSIS and grey based fuzzy inference system coupled with Taguchi method to optimize quality and productivity measurements in CNC milling operation. The optimization of machining parameters has been carried out with minimum number of test conditions by using orthogonal array. Based on experimental results and data analysis, the following conclusions are summarized below:

- (1) Maximization the MRR and minimization the surface roughness has been found possible simultaneously under this aforesaid optimal parameter combination.
- (2) Multiple objectives can be optimized in an effective logical efficient manner.
- (3) This method can efficiently be applied in any manufacturing/ production environment to determine the optimal environment capable of producing desired yield either in the process or in the product.

**Table 1:** Design of experiment

Sl. No.	Factorial combination (Coded form)		
	N	f	d
1	1	1	1
2	1	2	2
3	1	3	3
4	2	1	2
5	2	2	3
6	2	3	1
7	3	1	3
8	3	2	1
9	3	3	2

**Table 2:** Domain of experiments

Factors	Unit	Level 1	Level 2	Level 3
Cutting Speed, N	RPM	750	1000	1500
Feed Rate, f	mm/min	50	150	200
Depth of Cut, d	mm	0.2	0.4	0.6

**Table 3:** Experimental data

Sl. No.	Experimental Data	
	MRR (mm <sup>3</sup> /min)	R <sub>a</sub> (μm)
1	124.78418	3.93
2	536.22591	2.22
3	1371.11210	2.66
4	183.44741	3.8
5	924.55930	3.0
6	360.68593	3.133
7	375.26527	3.066
8	694.62268	3.2
9	1231.74102	3.133

**Table 4:** Normalized data for MRR and R<sub>a</sub> (grey relation generation)

Sl. No.	Normalized data	
	MRR	R <sub>a</sub>
1	0	0
2	0.33012	1
3	1	0.74269
4	0.04706	0.07602
5	0.64170	0.54386
6	0.18927	0.46608
7	0.20097	0.50526
8	0.45721	0.42690
9	0.88817	0.46608

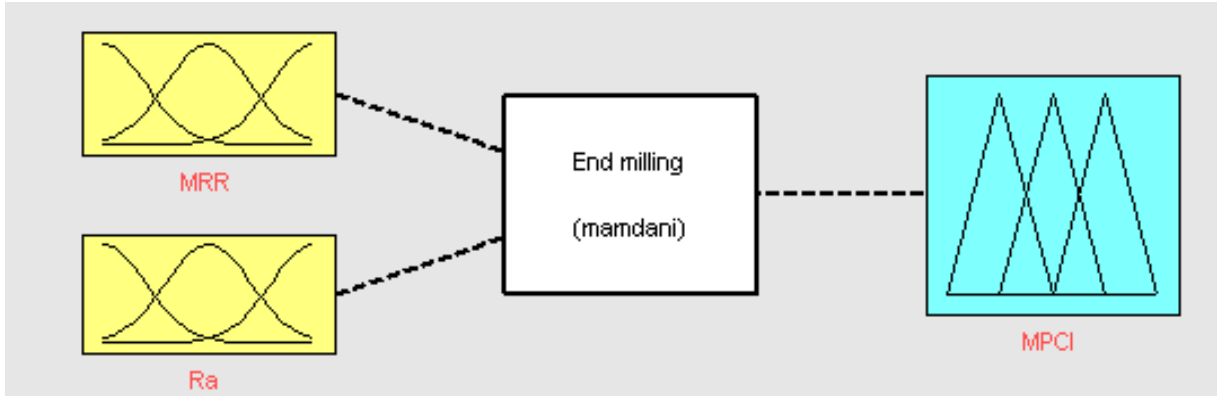
**Table 5:** Quality loss estimates ( $\Delta_{oi}$ ) of individual responses

Sl. No.	$\Delta_{oi}$ for MRR	$\Delta_{oi}$ for R <sub>a</sub>
1	1	1
2	0.66988	0
3	0	0.25731
4	0.95294	0.92398
5	0.35830	0.45614
6	0.81073	0.53392
7	0.79903	0.49474
8	0.54279	0.57310
9	0.11183	0.53392

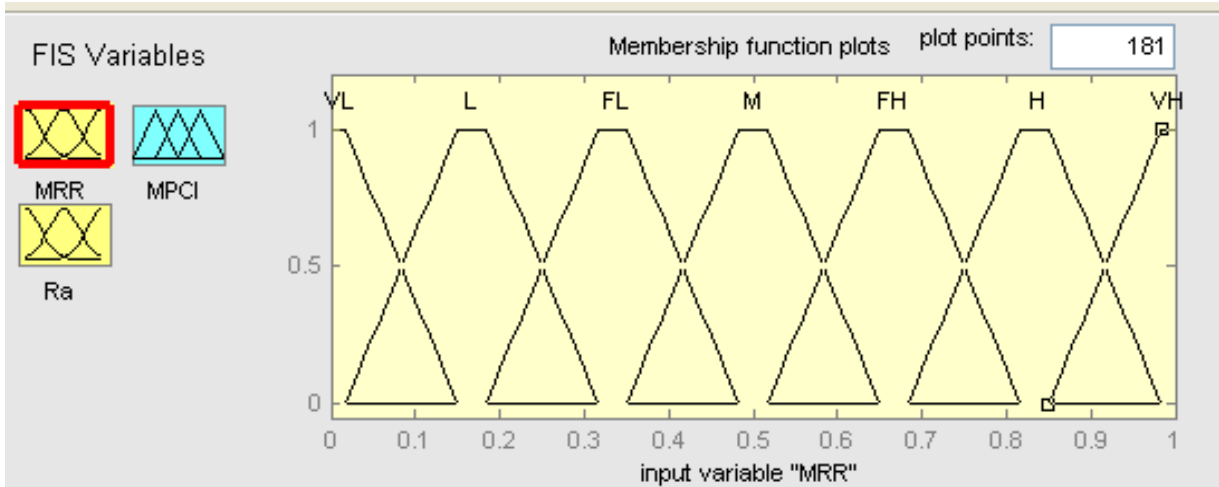


**Table 6:** Grey relational coefficient  $[\xi_i(k)]$  of individual responses  
[Distinguishing coefficient ( $\psi$ ) value taken as 0.5]

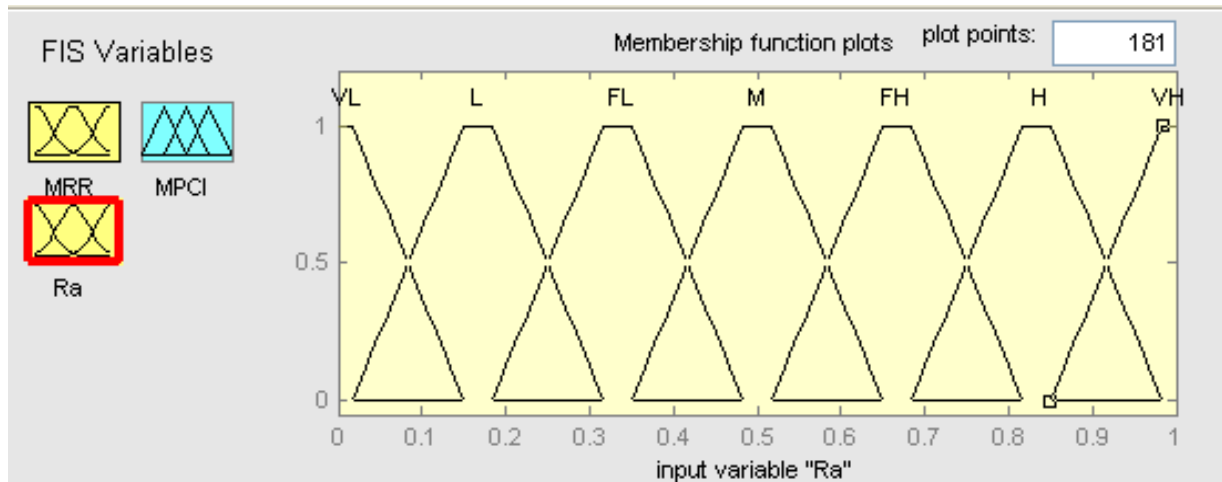
Sl. No.	$\xi_i(k)$ of MRR	$\xi_i(k)$ of Ra
1	0.33333	0.33333
2	0.42739	1
3	1	0.66023
4	0.34413	0.35112
5	0.58254	0.52293
6	0.38146	0.48359
7	0.38490	0.50264
8	0.47948	0.46594
9	0.81722	0.48359



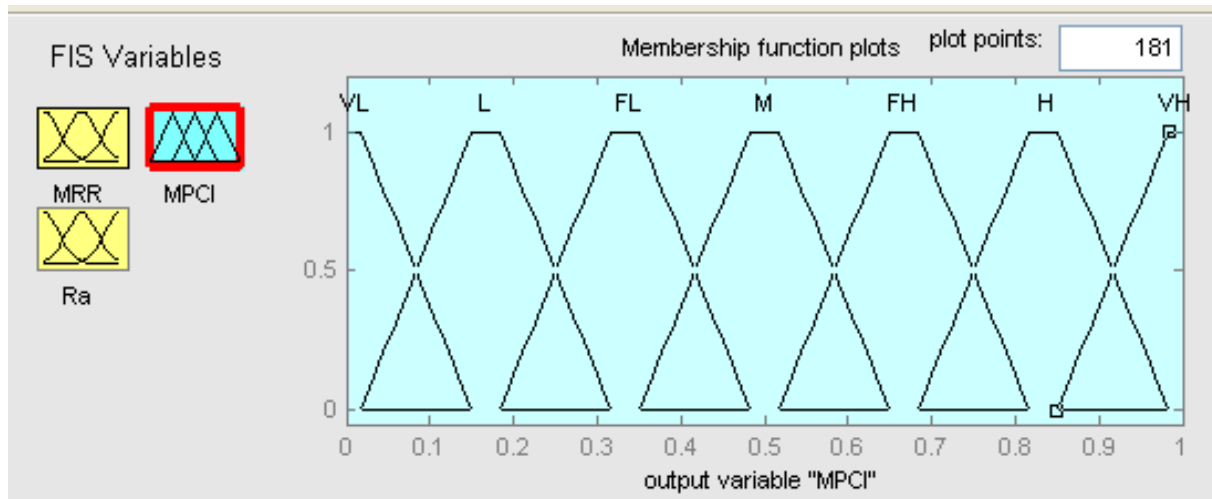
**Fig. 1** Proposed Fuzzy Inference System (FIS)



**Fig. 2** Membership Functions (MFs) for MRR



**Fig. 3** Membership Functions (MFs) for  $R_a$



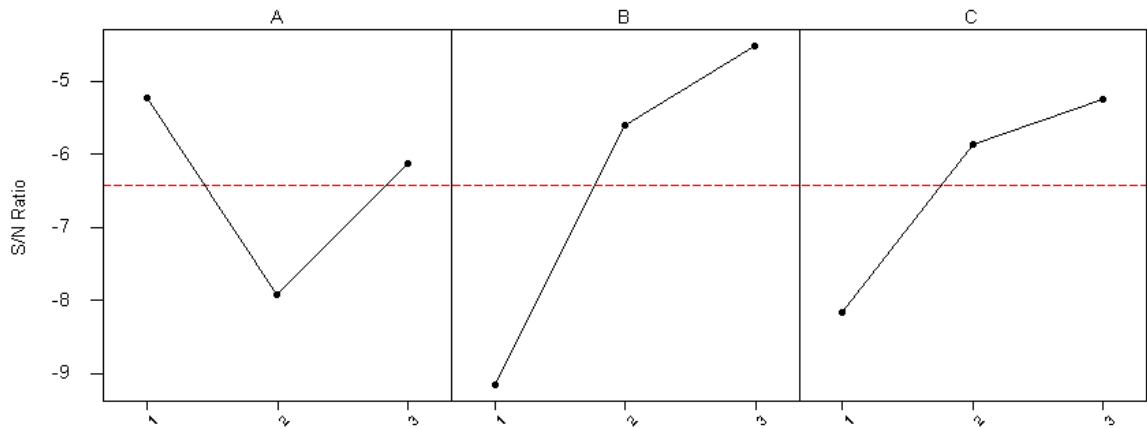
**Fig. 4** Membership Functions for MPCl

**Table 7:** Fuzzy rule matrix

MPCl		MRR						
		VL	L	FL	M	FH	H	VH
$R_a$	VL	VL	VL	L	L	FL	FL	M
	L	VL	VL	L	FL	FL	M	M
	FL	L	L	FL	FL	M	M	FH
	M	L	L	FL	M	M	FH	H
	FH	L	FL	FL	M	FH	H	H
	H	L	FL	M	FH	FH	H	VH
	VH	FL	FL	M	FH	H	H	VH

**Table 8:** Computed MPCl values

Sl. No.	MPCI
1	0.333
2	0.595
3	0.833
4	0.333
5	0.515
6	0.378
7	0.382
8	0.473
9	0.667

**Fig. 5** S/N ratio plot for MPCIs (Evaluation of optimal setting) **A1 B3 C3/ N1 f3 d3****Table 9:** Response Table for Signal to Noise Ratios

Level	A	B	C
1	-5.21596	-9.15365	-8.16802
2	-7.92171	-5.59210	-5.85942
3	-6.12633	-4.51825	-5.23656
Delta	2.70575	4.63541	2.93146
Rank	3	1	2

**Table 10:** Normalized decision-making matrix

Sl. No.	Normalized Response Data	
	$R_a$	MRR
1	0.414092	0.053989
2	0.231807	0.232001
3	0.280276	0.59322
4	0.400395	0.079366
5	0.316101	0.400016
6	0.330115	0.156053
7	0.323055	0.162361
8	0.337174	0.300533
9	0.330115	0.532921

**Table 11:** Weighted normalized decision-making matrix

Sl. No.	Weighted Normalized Response Data	
	$R_a$	MRR
1	0.207046	0.026994
2	0.115904	0.116001
3	0.140138	0.29661
4	0.200197	0.039683
5	0.15805	0.200008
6	0.165057	0.078027
7	0.161528	0.08118
8	0.168587	0.150266
9	0.165057	0.26646

**Table 12:** Computed Ideal and Negative-Ideal solutions

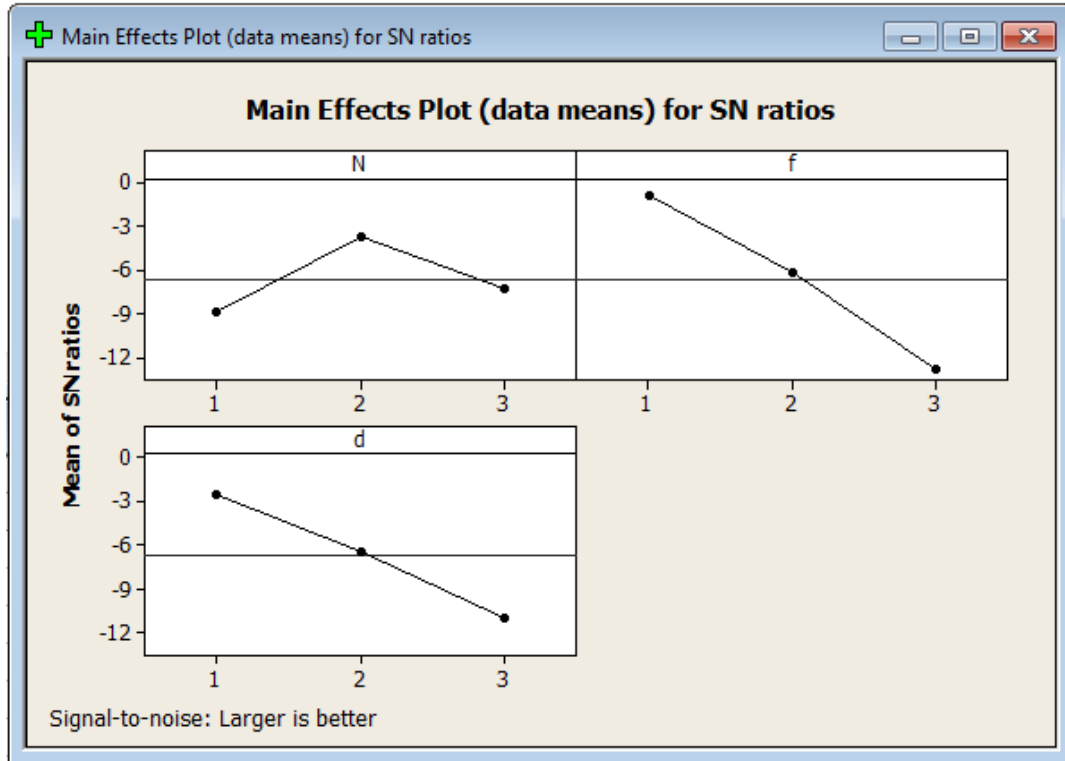
Sl. No.	Ideal	Negative-Ideal
1	0.115904	0.207046
2	0.29661	0.026994

**Table 13:** Computed distance measures

Sl. No.	S <sup>+</sup>	S <sup>-</sup>
1	0.284604	0
2	0.180609	0.127394
3	0.024234	0.277794
4	0.270401	0.014419
5	0.105396	0.179818
6	0.224041	0.066087
7	0.220208	0.070767
8	0.155538	0.129132
9	0.057663	0.243119

**Table 14:** Calculation of Closeness Coefficient (CC) against each experimental run

Sl. No.	C <sup>+</sup>	S/N Ratio
1	1.000000	0.0000
2	0.586387	-4.6363
3	0.080238	-21.9124
4	0.949375	-0.4512
5	0.369533	-8.6469
6	0.772214	-2.2452
7	0.756794	-2.4205
8	0.546380	-5.2501
9	0.191710	-14.3471



**Fig. 6** S/N ratio plot for CC (Evaluation of optimal setting) **N2 f1 d1**

**Table 15:** Mean-Response (S/N ratios of CC) table

Level	N	f	d
1	-8.8496	-0.9572	-2.4984
2	-3.7811	-6.1778	-6.4782
3	-7.3392	-12.8349	-10.9933
<b>Delta</b>	<b>5.0684</b>	<b>11.8777</b>	<b>8.4948</b>
<b>Rank</b>	<b>3</b>	<b>1</b>	<b>2</b>

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## Communication

1. Bikash Mohanty, Kumar Abhishek, Chitrasen Samantra, **Saurav Datta**, Siba Sankar Mahapatra, “*Use of GRA-Fuzzy towards Multi-Response Optimization in CNC End Milling*”, The 3rd Asian Symposium on Materials & Processing, ASMP2012, will be held from August 30 to 31, 2012 in IIT Chennai, India. (**Abstract Submitted**)